I. A TEST PROBLEM

We propose to solve a simple 1D problem, using a single domain. Let us consider the following equation:

$$u'' - 4u' + 4u = \exp(x) + C \quad ; \quad x \in [-1; 1] \quad ; \quad C = -\frac{4e}{1+e^2}. \tag{1}$$

For the boundary conditions, we adopt :

$$u(-1) = 0 \quad \text{and} \quad u(1) = 0. \tag{2}$$

Under those conditions, the solution of the problem is

$$u(x) = \exp(x) - \frac{\sinh 1}{\sinh 2} \exp(2x) + \frac{C}{4}. \tag{3}$$

II. SUGGESTED STEPS

- Construct the matrix representation of the differential operator.
- Solve the equation using one or more of usual methods : Tau, collocation and Galerkin.
- Check whether the methods are optimal or not.

III. DISCONTINUOUS SOURCE

Let us consider the following problem :

$$-u'' + 4u = S \quad ; \quad x \in [-1; 1] \tag{4}$$

$$u(-1) = 0 \quad ; \quad u(1) = 0 \tag{5}$$

$$S(x < 0) = 1 \quad ; \quad S(x > 0) = 0 \tag{6}$$

The solution is given by :

$$u(x < 0) = \frac{1}{4} - \left(\frac{e^2}{4} + B^- e^4\right) \exp(2x) + B^- \exp(-2x) \tag{7}$$

$$u(x > 0) = B^+ \left(\exp(-2x) - \frac{1}{e^4} \exp(2x)\right) \tag{8}$$

$$B^- = -\frac{1}{8(1+e^2)} - \frac{e^2}{8(1+e^4)} \tag{9}$$

$$B^+ = \frac{e^4}{8} \left(\frac{e^2}{(1+e^4)} - \frac{1}{(1+e^2)}\right) \tag{10}$$

IV. SUGGESTED STEPS

- Verify that Gibbs phenomenon appear when using a single domain method.
- Implement one or more if the multi-domain solvers (Tau, Homogeneous solutions or variationnal).
- Check that exponential convergence to the solution is recovered.